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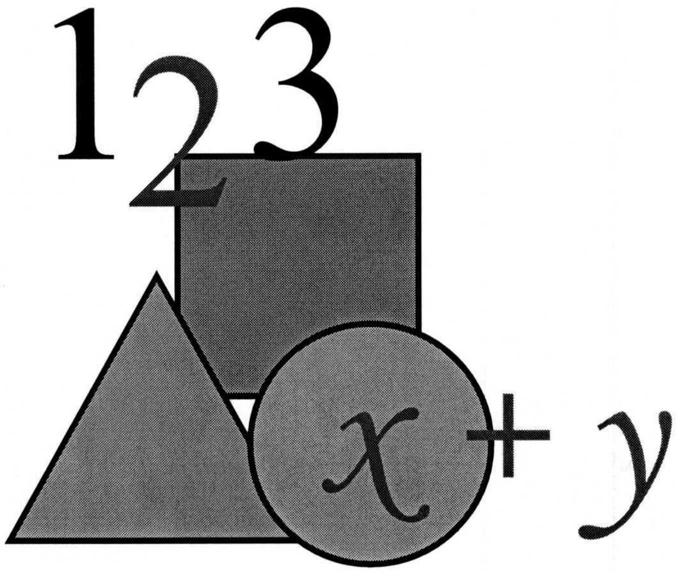
Number Sense to Algebra:



Fractions Using Tangrams

Building the Foundation With Conceptual Understanding

Shirley Roath
Administrator
District and School Success Center
sroath@rcoe.us

Linda Braatz-Brown
Director Special Programs
ALPHA Center, UC Riverside
linda.braatz-brown@ucr.edu

Tangram Equations
By Shirley Roath

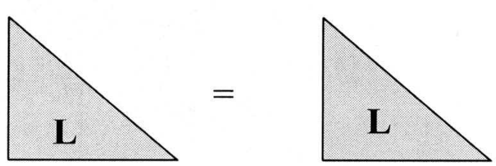
Teachers are being asked to include formal operations of algebra and algebraic properties into the mathematics curriculum at earlier grade levels. However some students are not developmentally prepared to deal with such abstract ideas. Difficulties arise when children are expected to memorize procedures and terms without concrete experiences that build their conceptual understanding. The result is that some students fall behind their classmates.

In this activity, students use tangram pieces to build equations in a concrete form, record their equations using variables, and use the properties of algebra to prove their equivalencies, then substitute values in order to prove their equations arithmetically. This is an excellent introduction to writing and solving equations and justification of the steps of an algebraic solution. It is also quite effective for remediation and intervention for fractions, properties of equations and substitution.

Supply each student with a set of tangrams. You can use the tangrams that students made themselves in the previous activity, student sets of tangrams that are sold commercially or use a dye cut machine to cut out the tangram pieces. Use the overhead or document camera to display the tangram pieces as you write "variable labels" on them with a washable marker. Explain to students that for the sake of time you are going to label the large triangles with an "L," the medium triangle with an "M," the small triangles with an "S," the parallelogram with a "P," and since the "S" has already been used an "R" for rectangle or rhombus on the small square. Students may want to write these labels on their tangram pieces also.

Remind students of the conditions for congruence, "figures with the exact same size and shape." Also remind students that figures can have the same area but have different shapes, (use examples from chapter 2 rectangles). Ask students to build congruent shapes using only the pieces from their own tangram. Model a few examples for them on the overhead or document camera and show them how they will be expected to record their unique arrangements.
Example 1:

Drawing (congruent) | **Equation (equal area)**



L = L

Example 2:

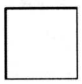
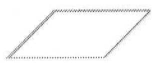
Drawing (congruent)	**Equation (equal area)**
	**S + S = M or 2S = M**

Have students work in groups to record their "tangram equations" on chart paper. They should have two columns as shown above: one column using drawings or actual tangram pieces glued down, the other with the variable abbreviations. An alternative is to have each student create their own drawings and equations using the *tangram equation worksheet.* Keep a class list (using variables) of unique equations and solutions. Introduce the properties of algebra as students share their equations. Two examples in dialogue form appear below.

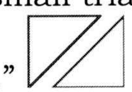
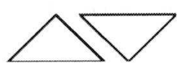
Student Talk Example 1:

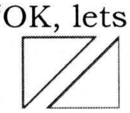
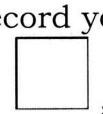
Student solution: "R = P"

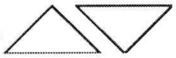
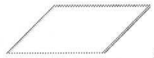
Teacher: "Can you show that this is a true statement?"

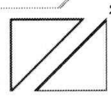
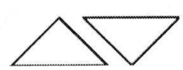
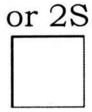
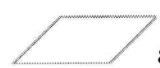
Student demonstrates:  = 

Teacher: "How do you know that they are equal? Can you prove it?"

Student demonstrates: "Two small triangles fit into the square and two small triangles also fit into the parallelogram."  

Teacher: "OK, lets record your thinking. You said that two small triangles fit into the square, so  = , or S + S = R. Could we say 2S = R? Then what?"

Student: " = , or 2S = P."

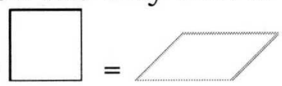
Teacher: "Good, now since  is the same as , or 2S = 2S we can substitute the square and parallelogram for the triangles. So  =  and we can write R = P."

This student used the substitution property of equality to prove her equation. Let's look at another property demonstrated using the same problem.

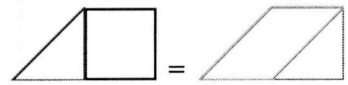
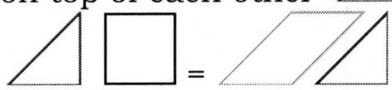
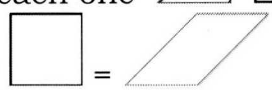
Student Talk Example 2:

Student solution: "R = P"

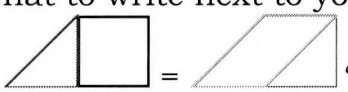
Teacher: "Can you show me why this is true?"

Student demonstrates: 

Teacher: "How do you know that they are equal? Can you prove it?"

Student demonstrates: "I can make two trapezoids that are the same and I can show you they are the same by stacking them on top of each other . Now if I take the triangle off of each one , I am left with the square and the parallelogram." 

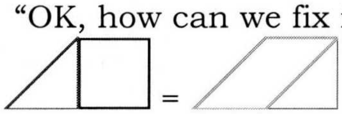
Teacher: "OK, let's record what you said with variables to go with your pictures. You tell me what to write next to your drawings."

Student:  "SR = PS."

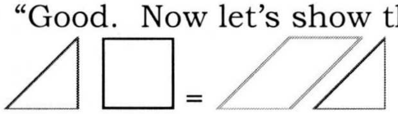
Teacher: "What operation are you asking me to do when you write letters or numbers and letters next to each other?"

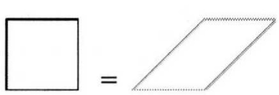
Student: "Multiplication."

Teacher: "OK, how can we fix it?"

Student:  "S + R = P + S."

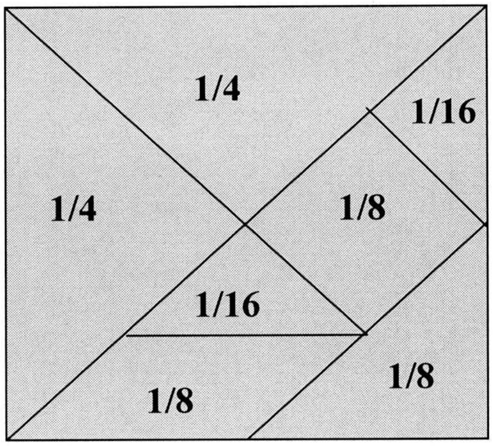
Teacher: "Good. Now let's show that we took the square off of both shapes."

Student:  "S + R - S = P + S - S."

 "R = P."

This student used the subtraction property of equality. Note that in both examples shown above the student begins with congruent shapes, but the final proof is that of equal areas of non congruent shapes.

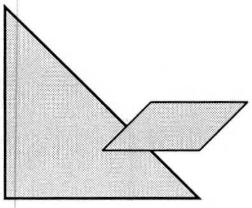
The last part of the *Tangram Equation Activity* involves substituting values into the student created equations in order to prove arithmetically that the areas are equal. In a previous activity, *Making Tangrams*, students divided a square sheet of paper with an area of one square unit into the seven tangram pieces and proved the fractional value of each. These areas were:



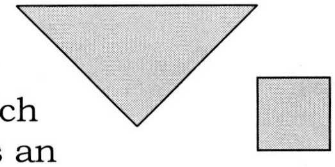
Using the last example:	S + R = P + S
Substitute the values.	1/16 + 1/8 = 1/8 + 1/16
Find common denominators.	1/16 + 2/16 = 2/16 + 1/16
Add.	3/16 = 3/16

Students should prove each of the equations that were listed on their chart paper or worksheet.

Algebra can be confusing and easily forgotten when presented to students as a list of rules, properties and procedures. This activity serves as a fun and concrete introduction to algebraic equations while reinforcing the connection between algebra and geometry.



Tangram Equations

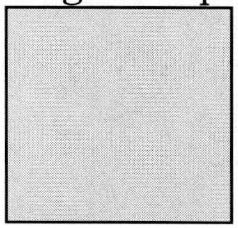
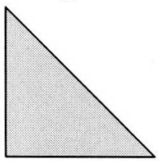
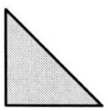
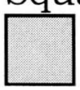
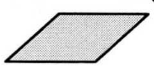


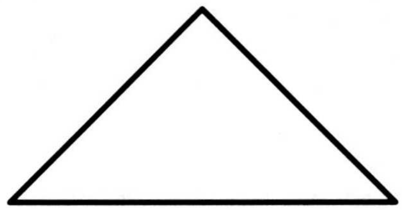
Using the tangram pieces create and sketch congruent shapes then record the areas as an equation. Draw the pieces with equal areas and write the equation.

Drawings	**Equations**

Making Tangrams

The value of the original square has been changed. Give the new values for each of the tangram shapes.

Original Square 	Original Square Area = 2	Original Square Area = ½	Original Square Area = .75
Large Triangle 	Large Triangle Area =	Large Triangle Area =	Large Triangle Area =
Medium Triangle 	Medium Triangle Area =	Medium Triangle Area =	Medium Triangle Area =
Small Triangle 	Small Triangle Area =	Small Triangle Area =	Small Triangle Area =
Square 	Square Area =	Square Area =	Square Area =
Parallelogram 	Parallelogram Area =	Parallelogram Area =	Parallelogram Area =

If  = ¼

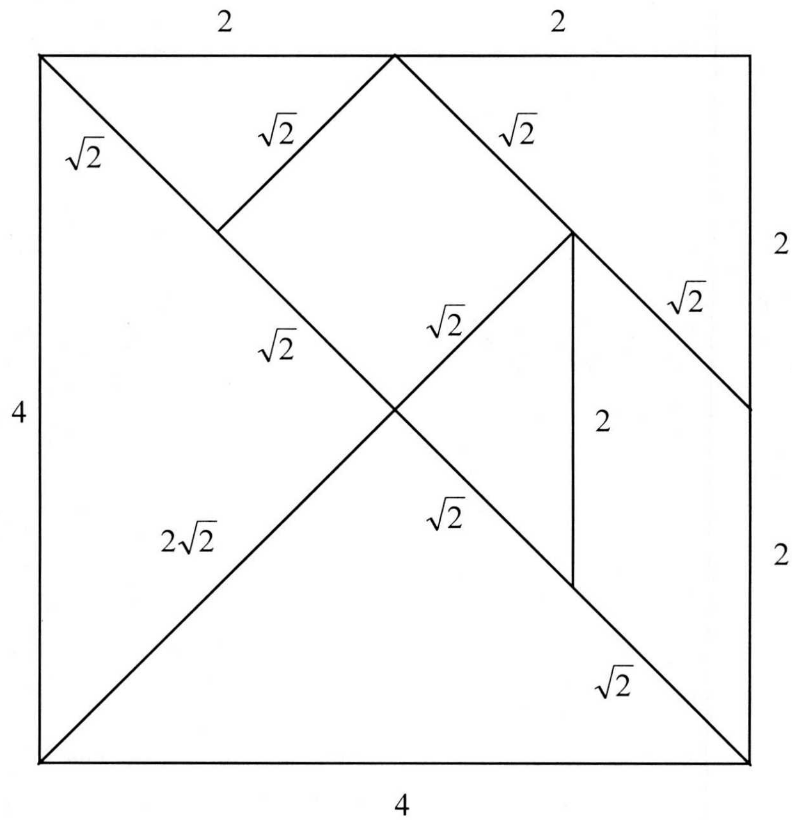
form a quadrilateral

equal to 2¼.

Prove it!

Think of each tangram piece as a fractional part. Have 2 sets of tangrams available!

According to Chinese legend, tangrams were created when a man dropped a porcelain tile that broke into seven pieces. When he tried to reassemble the tile, he found that he could create hundreds of different shapes.



Use all seven tangram pieces to create the indicated figure, then find the perimeter of the figure.